DEVELOPMENT OF PERTURBATIONS NEAR A SURFACE IN A SUPERSONIC FLOW

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An understanding of the problem of the transition from a laminar to a turbulent boundary layer requires study of the development of small perturbations. In experimental approaches to the problem, investigators employ hot-wire anemometers with wire sensors located inside the boundary layer at both subsonic and supersonic velocities. The use of such sensors in the case of high supersonic velocities is more difficult for several reasons: high stagnation temperature, the relatively high degree of contamination of the flow, the relatively high velocity head, etc. Under these conditions, it seems preferable to study the development of waves directly on the surface of the model. Here, the perturbations can be detected with film-type thermoanemometric sensors (such as are used to measure shear stresses) and pressure gauges normally used in studies of turbulent boundary layers. In [1], pressure gauges were used to record unstable waves in a supersonic laminar boundary layer.

From the viewpoint of the theory of stability of parallel flows, the wave parameters obtained from measurements inside the boundary layer and on the surface should coincide. In this regard, the degree of intensification of the perturbations can be altered by any type of sensor with sufficient sensitivity and resolution. For example, this quantity might be measured with a pitot tube placed on the surface of the model. Unfortunately, due to the non-parallelism of the flow in the boundary layer, the parameters of the wave are different at different distances from the surface. In [2], complete information was presented on the development of perturbations of mass rate, but no representations were given regarding the behavior of oscillations of other parameters, especially on the surface of the model.

Here, the development in a boundary layer of perturbations of all basic flow parameters is theoretically investigated. The results of calculations of the degree of reinforcement of the mass-rate gradient, the temperature gradient, and the pressure on the surface of a plate are compared to experimental data obtained with wire thermoanemometric sensors positioned on the model surface.

1. A method of calculating the development of perturbations on the basis of the linearized Navier–Stokes equations was described in [3] with allowance for the slight non-parallelism of the main flow. In accordance with the theory of stability of weakly non-parallel flows, the dependence of the perturbations on the dimensionless coordinates and time was taken in the form

$$Q_k(\bar{x}, \bar{y}) = A(\bar{x})\, q_k(\varepsilon\bar{x}, \bar{y}) \exp\left[i\left(\int\limits_{\bar{x}_0}^{\bar{x}} \alpha(\xi)\, d\xi + \beta\bar{z} - \omega t\right)\right]. \quad (1.1)$$

Here, $A(\bar{x})$ is the amplitude; q_k is the eigenfunction of the theory of stability of locally parallel flows, it being parametrically dependent on the longitudinal coordinate $\bar{x}$; ε is a small parameter characterizing the non-parallelism of the flow; α and β are wave numbers; ω is the angular frequency. The vector function q_k is normalized so that the maximum flow stresses in the plane $(\bar{x}, \bar{z})$ are independent of $\bar{x}$. The space coordinates were made dimensionless by means of the formula $\delta = \sqrt{x\nu_e/u_e}$, while time was made dimensionless in accordance with $\tau_1 = \delta/u_e$ (the subscript e denotes parameters of the flow at the boundary of the boundary layer). All of the calculations were performed for a boundary layer on a flat plate in a flow of air with the adiabatic exponent $\gamma = 1.4$, the Prandtl number Pr = 0.72, and a temperature dependence of viscosity in accordance with the Sutherland law.

In conformity with (1.1), the degree of increase in the perturbation of the k-th parameter of the flow is written in the form

$$-2\bar{\alpha}_i^k = \frac{1}{|Q_k|}\frac{d|Q_k|}{d\bar{x}} = \mathrm{Real}\left[2\alpha i + \frac{1}{A}\frac{dA}{d\bar{x}} + \frac{1}{q_k}\frac{\partial q_k}{\partial \bar{x}}\right], \quad (1.2)$$

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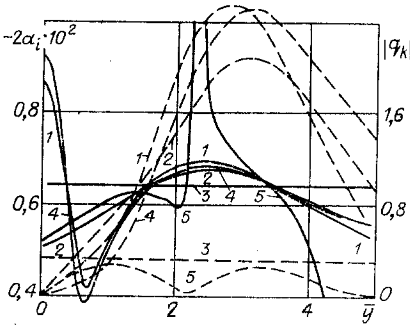

Fig. 1

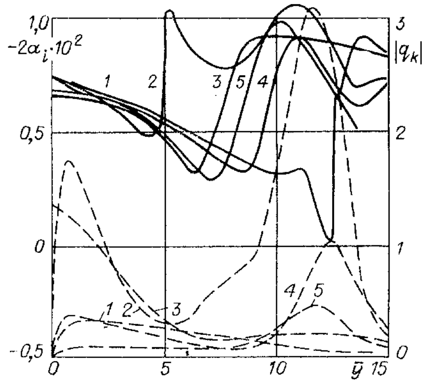

Fig. 2

where $\alpha = \alpha_r + i\alpha_i$ is the wave vector in the zeroth approximation of the theory of stability of weakly nonparallel flows (in the present case, α is the eigenvalue of the Dan–Lean problem [4]). The second term of the sum characterizes the effect of small terms in the complete stability equations, including those associated with a change in the amplitude of the perturbations. Finally, the third term directly accounts for the change in amplitudes with the prescribed normalization and depends not only on $\bar{x}$, but also on the coordinate $\bar{y}$ normal to the plate.

The thermoanemometric sensor reacts to fluctuations of mass rate and stagnation temperature. The greater the heating of the sensor, the greater the contribution of mass-rate fluctuations to the total signal of the sensor. Inside the boundary layer, with substantial heating of the sensor taking place, the latter is sensitive mainly to mass-rate fluctuations. If these fluctuations are made dimensionless relative to the mass rate on the external boundary of the boundary layer, then they can be written in the form of a linear combination of the perturbations of velocity $f = u'/u_e$, temperature $\vartheta = T'/T_e$, and pressure $\pi = p'/p_e$:

$$m = (\rho u)'/\rho_e u_e = f/T + (\pi - \vartheta)u/T \qquad (1.3)$$

(T and u are the dimensionless distributions of temperature and velocity over the boundary layer).

The corresponding value of the oscillations of stagnation temperature is determined from the expression

$$\vartheta_0 = (\vartheta + (\gamma - 1)M^2\bar{u}f)/(1 + (\gamma - 1)M^2/2). \qquad (1.4)$$

Figure 1 shows the results of calculations of the amplitudes of perturbations (dashed curves) of longitudinal velocity 1, temperature 2, pressure 3, mass rate 4, and stagnation temperature 5 and the degrees of their increase (solid lines) for a Mach number M = 2 and Reynolds number $R = \sqrt{xu_e/\nu_e} = 720$. The angle of the normal of the wave front to the flow direction $\chi = 60°$, while the frequency parameter $F = \omega\nu_e/u_e^2 = 0.36\cdot 10^{-4}$. It is evident that the amplitudes of the fluctuations of mass rate, velocity, and temperature are comparable in magnitude and are similar in character with regard to their dependence on $\bar{y} = y/\sqrt{x\nu_e/u_e}$. As a result, the behavior of their increases is also similar over much of the boundary layer. A difference is seen only near the surface. For the perturbations of velocity and mass rate, the degree of their increase near the wall is greater than the corresponding values in the regions of the maximum amplitudes. Conversely, the degree of reinforcement of the temperature perturbations near the surface is lower than near the amplitude maximum. For the layer as a whole, the level of the pulsations of stagnation temperature is considerably lower than the level of the mass-rate pulsations and becomes greater than the latter only at $\bar{y} \leq 1$. The intensity of the pressure oscillations and the degree of their reinforcement are constant within the investigated range of $\bar{y}$. The level of the pressure perturbations in the region of the mass-rate maximum is substantially lower than the fluctuations of longitudinal velocity and temperature. Thus, the contribution of pressure perturbations to the mass-rate oscillations may be significant only near the surface, where f and ϑ are small. The degree of intensification of the pressure oscillations is close in value to the corresponding values for the other parameters near their maxima and is equal to $0.64\cdot 10^{-2}$.

The data shown in Fig. 1 were obtained for the first unstable mode of oscillation. According to [5], with an increase in M, higher modes with $\chi = 0$ begin to predominate in the development of instability. Figure 2 shows the results of calculations for the second mode with $M = 4.5$, $R = 1550$, $F = 0.135 \cdot 10^{-4}$, and $\chi = 0$ (the notation is the same as in Fig. 1). A characteristic difference from Fig. 1 is the marked change in the amplitude of the pressure perturbations across the layer. Two maxima are seen in its distribution: one coincides with the position of the peak in the distribution of mass-rate perturbations, while the second (and larger) maximum is located on the surface of the plate. There are two temperature-fluctuation maxima. The largest is seen near the peak of the mass-rate. The fluctuations of longitudinal velocity reach their greatest values near the wall. Their absolute value is considerably lower than the maximum fluctuations of pressure and temperature. The level of the stagnation-temperature fluctuations is lower than the level of the mass-rate fluctuations at $\bar{y} > 10.5$ but higher at $\bar{y} \leq 7$. The difference is substantial at $\bar{y} < 5$.

With allowance for the distributions of the perturbations of the flow parameters inside the layer and Eq. (1.3), it is evident that the greatest fluctuations of mass rate (at $\bar{y} \approx 12.5$) are determined by temperature fluctuations. The relations showing the degree of intensification of the perturbations are complex functions of the coordinate $\bar{y}$. This also applies to the pressure perturbations, in contrast to the data shown in Fig. 1. However, the intensifications of the maximum perturbations differ little from one type of perturbation to the next. It is important to note that the rate of intensification of the perturbations of all of the parameters is roughly the same near the wall and is close to the values obtained near the maxima of the perturbations. Moreover, the theory of plane-parallel flows, based on the Dan–Lean equations [4], gives $-2\alpha_i \approx 0.8 \cdot 10^{-2}$ for this case. The latter value is close to the values (see Fig. 2) near the wall and the perturbation maxima.

Let us discuss the development of perturbations near the surface in somewhat greater detail. Since the fluctuations of temperature, stagnation temperature, velocity, and mass rate on the wall are equal to zero, their logarithmic derivatives cannot be calculated directly. On the other hand, it can be shown that they coincide with the rates of increase in the corresponding derivatives with respect to $\bar{y}$. For $M = 2$, we obtained the following values: $-2\alpha_i^{m}(0) = 0.92 \cdot 10^{-2}$, $-2\alpha_i^{f}(0) = 0.87 \cdot 10^{-2}$, $-2\alpha_i^{\vartheta}(0) = -2\alpha_i^{\vartheta_0}(0) = 0.52 \cdot 10^{-2}$, $-2\alpha_i^{\pi}(0) = 0.64 \cdot 10^{-2}$, while we obtained $-2\alpha_i = 0.55 \cdot 10^{-2}$ from the Dan–Lean equation. As already noted above, the degree of intensification of the perturbations of the different parameters is similar for the second mode of oscillation with $M = 4.5$, to wit: $-2\alpha_i^{m}(0) = 0.68 \cdot 10^{-2}$, $-2\alpha_i^{f}(0) = 0.75 \cdot 10^{-2}$, $-2\alpha_i^{\vartheta}(0) = -2\alpha_i^{\vartheta_0}(0) = 0.75 \cdot 10^{-2}$, $-2\alpha_i^{\pi}(0) = 0.67 \cdot 10^{-2}$. Thus, the degree of intensification of the perturbations for the second mode of oscillation near the wall can be determined from any of the hydrodynamic quantities. The value obtained here is close to that found from the Dan–Lean equations: $-2\alpha_i = 0.8 \cdot 10^{-2}$.

Due to the large difference in the values of $\alpha_i^{qk}(0)$, checking the theory of stability of parallel flows by experimental study of perturbations of velocity, mass rate, and temperature near the surface is problematic – at least for the first mode of oscillation and small Reynolds numbers. Pressure fluctuations are most suitable in this regard. For both the second mode with large R and the first mode (even with moderate R), the degree of intensification of pressure perturbations near the wall agrees well with data from the theory of parallel flows. It should be noted, however, that the level of the pressure fluctuations is low at low M, and highly sensitive gauges will be needed for these purposes.

2. Experiments were conducted in a T-325 wind tunnel at the ITPM of the Siberian Department of the Soviet Academy of Sciences [6]. The tube has a working part with a cross section of 200 × 200 mm (the turbulence characteristics of the flow in the working part of the T-325 were given in [7]). The tests were conducted at $M = 2$ and 4, the stagnation temperature $T_0 \approx 300$ K, and $Re_1 = (u/\nu)_\infty = (4.7\text{-}30) \cdot 10^6$ m^{-1} (Re_1 is the unit Reynolds number). The flow parameters (the temperature in the precombustion chamber, the static pressure in the working part, and the total pressure in the precombustion chamber) were determined with the standard equipment for a wind tunnel. This allowed us to find Re_1 with a standard deviation no greater than ±1.5%. The model in the experiment was a flat steel plate 450 mm long, 200 mm wide, and 8 mm thick with a leading-edge blunting of 0.03-0.04 mm bevelled at 20°. The rms microroughness of the working surface was no greater than 2-3 μm. The plate was rigidly affixed to the side walls of the working part of the tunnel and was set at a zero angle of attack.

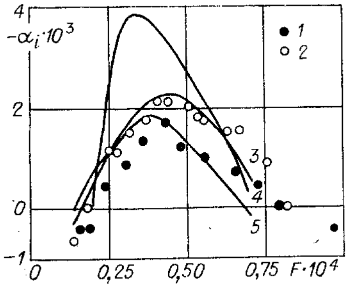

Fig. 3

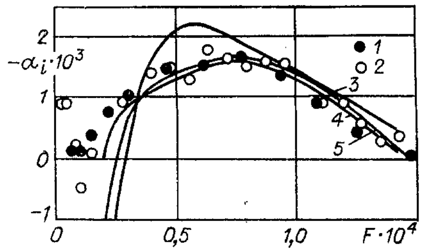

Fig. 4

The stability of the boundary layer and the position of the transition were determined with a TPT-3 dc thermoanemometer and five thermoanemometric pitot tubes located on the surface of the model (on ebony bases) 10 mm from each other. The ebony bases were set in the surface so as to be flush with it. We used sensors with a gold-plated tungsten wire 6 μm in diameter and 1.5 mm in length. All of the sensors had nearly the same resistance (the maximum difference did not exceed 1%), which allowed us to consider them to be identical.

We used U2-8 selective amplifiers, a V6-9 selective microvoltmeter, and a G3-112/1 signal generator for the amplitude-frequency analysis of the signal obtained from the thermoanemometer. The degree of intensification of the perturbations was determined from the relation $\alpha_i = -0.5d(\ln Q f_1)/dR$ (Q_{f_1} is the amplitude of the perturbation of dimensional frequency f_1). The dimensionless frequency $F = 2\pi f_1/(\mathrm{Re}_1 u_\infty)$.

The position of the transition was determined with one surface sensor from the change in Re_1 in the range $(16\text{-}30)\cdot 10^6$ m^{-1} on the basis of analysis of the dependence of the rms signal of the thermoanemometer $\langle e \rangle$ on Re_1. The relative error of R and F was ±2% and ±2.5%, while the absolute error of α_i was $\pm 0.3\cdot 10^{-3}$.

Using the amount of heating of the sensor by the air flow per unit time, we determined the energy $\int_s q ds$, where s is the area of the working surface of the sensor, while the heat flux $q = \lambda dT/dr = \bar{q} + q'$ ($\bar{q}$ is the mean heat flux, q' is the fluctuation heat flux, λ is the thermal conductivity, and r is the coordinate normal to the surface of the heated element of the sensor). For a film-type surface sensor, $q = \lambda dT/dy$, $q' = \lambda' d\bar{T}/dy + \bar{\lambda} dT'/dy = (d\bar{T}/dy)d\bar{\lambda}/d\bar{T}T' + \bar{\lambda} dT'/dy$. If the base had had the same heat-conducting properties as the steel, then on the base (as on the wall of the model) $T'(0) \approx 0$ (see Figs. 1 and 2 [8]) and $q'(0) \approx \bar{\lambda} dT'/dy$. Then we could have used the change in this quantity along the longitudinal coordinate to find stability characteristics of the boundary layer on the basis of data obtained from unheated film-type thermoanemometers. For the tiny surface sensor, the relation for the fluctuations of heat flux is a linear combination of $\bar{\lambda} dT'/dy$ and the term accounting for the gradient (with respect to y) of the mass-rate fluctuations (the contributions of these terms are different for different flow conditions and wall-material properties).

The first series of tests was conducted with M = 2 and R = 685 ($\mathrm{Re}_1 = 4.7\cdot 10^6$ m^{-1}). The results (points 1) are shown in Fig. 3 in the form of the dependence of the degree (rate) of increase in the perturbations $-\alpha_i$ on F. These results are compared with data from [9] (points 2), where the stability of the boundary layer at M = 2, R = 660 and 648 was determined in a layer that was close to "critical" (in a layer with maximal fluctuations of stress on the wire of the thermoanemometer). Also shown are the results of calculation of the growth of perturbations for maximum (in the boundary layer) fluctuations of mass rate (line 3), for fluctuations of the mass rate on the wall (line 4), and for fluctuations of temperature and stagnation temperature on the wall (here, they coincide) (line 5) at M = 2, R = 700, and $\chi = 42°$.

The second series of tests was conducted with M = 4 and R = 795 ($\mathrm{Re}_1 = 6\cdot 10^6$ m^{-1}). The measurements (points 1 in Fig. 4) are compared with data from [10, 11] (points 2) for M = 4, R = 780, and $\chi = 46°$ found near the critical layer (the notation for lines 3-5 is the same as in Fig. 3).

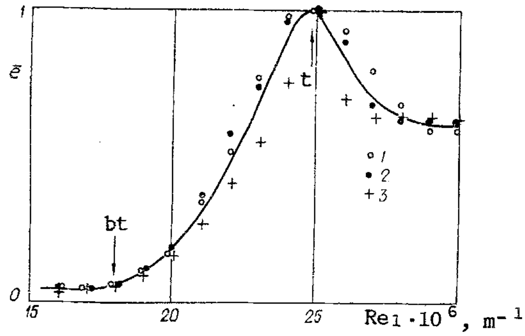

Fig. 5

It is evident from Figs. 3 and 4 that whereas the empirical relations 1 and 2 differ somewhat from each other for M = 2 (the degree of intensification is somewhat lower on the wall), there is almost no difference for M = 4 (it is within the measurement error). It is also evident that curve 5 "increases" relative to curves 3 and 4 with an increase in M. In accordance with this, there is also an increase in the ratio of the empirically established degrees of intensification of perturbations on the wall to the corresponding values in the near-critical layer. The latter is evidence that temperature fluctuations do make some contribution to the total signal from the surface thermoanemometer. This conclusion was confirmed in additional tests in which we determined the degree of intensification of perturbations on the wall with different amounts of heating of the anemometer wire. With an increase in heating (corresponding to a relative increase in the sensitivity of the sensor to fluctuations of stagnation temperature), the degree of intensification of $-\alpha_i$ increased somewhat.

In the next series of experiments, with M = 2, we used a single surface sensor to find the position of the laminar-turbulent transition in the boundary layer (Fig. 5). In these tests, we varied the unit Reynolds number within the range $Re_1 = (16\text{-}30)\cdot 10^6$ m^{-1}, while $\bar{e}$ was normalized with respect to its maximum value in the rms signal from the anemometer (points 1-3 correspond to different amounts of heating of the wire of the sensor: 0.7, 0.4, 0.2). The position of the transition (t), determined from the maximum of the relation $\bar{e}(Re_1)$, corresponds to $Re_1 = 24.8\cdot 10^6$ m^{-1} and $Re_t = 2.73\cdot 10^6$. The beginning of the transition (bt), determined from the first appreciable increase in the strength of the signal $\bar{e}(Re_1)$, corresponds to $Re_1 = 18.2\cdot 10^6$ m^{-1} and $Re_t{}^b = 2.0\cdot 10^6$; the latter, determined in the present study with a thermoanemometer, is very close to $Re_t{}^b = 2.1\cdot 10^6$ obtained by means of a pitot tube [12] with the same flow parameters ($M = 2$, $Re_1 = 18\cdot 10^6$ m^{-1}). This shows that it is possible to reliably find the position of the transition with a surface thermoanemometer.

Thus, the entire perturbed flow field in the supersonic boundary layer has been calculated theoretically. The theory results agree well with experimental data on the degree of intensification of perturbations in the flow. The data obtained here with surface anemometers was roughly the same as the data obtained with a thermoanemometer placed inside the boundary layer (near the critical layer). However, the data obtained with surface sensors is less accurate, since the fluctuations of mass rate and stagnation temperature are smaller in the wall region than inside the layer. The stability of the flow near the wall can be more reliably and more conveniently determined in tests from the pressure fluctuations. At the same time, surface thermoanemometers can reliably determine the position of the transition of the boundary layer from laminar to turbulent flow.

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STABILITY OF A HIGH-SPEED BOUNDARY LAYER

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At present a direct relationship between development of turbulence and loss of stability of an initially laminar boundary layer is generally accepted. The qualitative effect of various factors on the position of the point of transition from a laminar form of motion to turbulent is on the whole predicted correctly by stability theory. This has been confirmed by results of many studies at infrasonic and moderately supersonic (Mach number M = 2-5) flow velocities. However, at higher supersonic velocities (M > 5) experimental studies of boundary layer stability are very few in number and have all been performed with one and the same AEDC/B aerodynamic tube (at The Arnold Center) (see, e.g., [1]).

1. The present study will theoretically and experimentally investigate the stability of a boundary layer at high supersonic flow velocities. The experiments were performed in the T-327A nitrogen tube of the Institute of Theoretical and Applied Mechanics of the Siberian Branch, Academy of Sciences of the USSR at Reynolds number $(Re_1)_\infty = (u/\nu)_\infty = (0.7\text{-}1)\cdot 10^6$ 1/m, forechamber braking temperature T_0 = 1100-1260 K, and pressure $p_0{}^\Phi = (11.6\text{-}13.2)\cdot 10^6$ Pa. The nitrogen purity level was 10 molecules of oxygen per million nitrogen molecules.

The working model was a polished steel plane plate 330 mm long and 8 mm thick, having the form of a trapezoid (nose width 62, trailing edge width, 32 mm). The leading edge was beveled at an angle of 7° and blunted to b = 1 mm. A copper—constantan thermocouple was used to measure model surface temperature in the region where boundary layer stability characteristics were determined. The surface temperature changed only slightly — by 2%. Its mean value $T_{(w)}$ = 297 K. Because of the temperature change the braking temperature factor varied over the range $T_w = T_{(w)}/T_{aw}$ = 0.28-0.32 (where T_{aw} is the reconstruction temperature). The plate was installed in two positions — at $\omega_0 = 0$ (the plate regime) and $\omega_0 = 7°$ (the wedge regime) (ω_0 is the angle of plate inclination relative to the unperturbed flow). Boundary layer stability was determined by a TPT-4 dc thermoanemometer. An amplitude—frequency analysis of the signals was also performed using U2-8 selective amplifiers, a V6-9 voltmeter, a GZ-112/1 signal generator, and filament-type thermoanemometric sensors

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